Cancellation of metrology gauge periodic nonlinearity by ACS-roll induced motion of delay line

RES

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1 Introduction

During SIM observations, the spacecraft will have some residual motion that is uncompensated by the Attitude Control System (ACS). This motion causes the length of the two interferometer arms to change even though the star is stationary. To keep the interferometer centered on the starlight fringe, the delay line moves, and this motion is monitored by metrology gauge beams.

The metrology gauges have a nonlinearity on the order of $\varepsilon = 3\,\mathrm{nm}$ with periodicity $\lambda/2$, and somewhat smaller errors with periodicity $\lambda/4$. These errors are reduced automatically by averaging the gauge signal as the delay line moves through several λ. We calculate the attenuation of the nonlinearity resulting from this averaging.

2 Motion and error suppression

Let x = delay line position, y = phasemeter output. The "one cycle per fringe" nonlinearity causes an instantaneous error $\delta = y - x$ given by:

$$\delta = \varepsilon \sin kx \tag{1}$$

where $k = 4\pi/\lambda$.

The error in an average of duration T, equal to the total time the white light fringe is tracked, is given by

$$\overline{\delta} = \varepsilon\overline{\sin kx} = \varepsilon A \tag{2}$$

where

$$A = \frac{1}{T}\int_0^T \sin kx(t)dt \tag{3}$$

is the factor by which the nonlinearity is suppressed. It is equivalent to the inverse of the "cyclic averaging" suppression factor resulting from frequency modulation. If for example, $\varepsilon = 3\,\mathrm{nm}$ and the allowed error is $30\,\mathrm{pm}$, then we require $A \lesssim 0.01$.

2.1 Delay line moves with constant velocity

Suppose that the delay line moves with constant velocity v. Then

$$A = \frac{1}{kvT}(1 - \cos kvT) = \frac{1}{kX}(1 - \cos kX) \tag{4}$$

where $X = vT$ is the total delay line motion during the measurement. The error has local maxima at $kX = \pi(2n+1)$, with n an integer. The attenuation is therefore poorest for $X = n\lambda/2 + \lambda/4$. Taking this worst case, the bound on A is

$$A < \frac{1}{2\pi n} \tag{5}$$

where $n = X/\lambda$ is the total motion in units of the wavelength.

2.2 Sinusoidal motion of delay line

The delay line will move sinusoidally if, for example, the ACS is underdamped:

$$x = X \sin \omega t. \tag{6}$$

Then

$$\sin kx = 2J_1(kX) \sin \omega t + 2J_3(kX) \sin 3\omega t + \dots \tag{7}$$

where the J_n are bessel functions. The J_3 term can be dropped without much error. Taking the average,

$$A = 2J_1(kX) \left[\frac{1 - \cos \omega T}{\omega T} \right] \tag{8}$$

For $p = kX > 1$ (corresponding to the relevant case of $X > 0.1$ micron), the contribution from the J_1 term is bounded by

$$|J_1(p)| < \frac{0.8}{\sqrt{p}}. \tag{9}$$

The term in [] brackets in Equation 8 is bounded by $2/(\omega T)$. Therefore the bound on A is

$$A < \frac{1.6}{\sqrt{kX}} \cdot \frac{2}{\omega T}. \tag{10}$$

Letting $n = X/\lambda$, the amplitude of the oscillation in wavelengths, and $m = \omega T/(2\pi)$, the number of cycles of oscillation,

$$A < \frac{0.14}{m\sqrt{n}}. \tag{11}$$